7.7 Fluid Pressure and Force

Objectives:

(1) Find fluid pressure and force.

Definition (Fluid Pressure)**.**
The **pressure** on an object at depth h is

$$\text{Pressure} = P = wh$$

where w is the weight-density of the liquid per unit of volume.

The fluid pressure at depth h is transmitted equally in all directions (Pascal's Principle). See figure 7.68 on page 507. Fluid pressure is given in terms of force per unit area ($P = F/A$). The fluid force on a submerged horizontal surface of area A is

$$\text{Fluid force} = F = PA = (\text{pressure})(\text{area})$$

Example.
Find the fluid force on a rectangular metal sheet measuring 3 feet by 4 feet that is submerged in 6 feet of water.

Solution.
The weight-density of water is 62.4 pounds per cubic foot. The sheet is 6 feet deep. The fluid pressure is

$$P = (62.4)(6) = 374.4 \text{ pounds per square foot.}$$

The area of the sheet is 12 square feet. The fluid force is

$$F = PA = \left(374.4 \frac{\text{pounds}}{\text{square foot}}\right) (12 \text{ square feet}) = 4492.8 \text{ pounds}$$

Suppose a vertical plate is submerged in a fluid of weight-density w per unit of volume. See figure 7.70 on page 508. To find the total force against one side of the plate from depth c to depth d, partition the interval $[c, d]$ into n subintervals of width Δy. Let y_i be a point in the y^{th} sub-interval. The length of the i^{th} rectangle is $L(y_i)$. The force against the rectangle is approximated by

$$\Delta F_i \approx w(\text{depth})(\text{area}) = wh(y_i)L(y_i)\Delta y$$

The force against the entire plate is approximated by

$$F \approx w \sum_{i=1}^{n} h(y_i) L(y_i) \Delta y$$

Let $n \to infty$ to set up a definite integral.

Definition.
The **force F exerted by a fluid** of constant weight-density w per unit of volume against a submerged vertical plane region from $y = c$ to $y = d$ is

$$F = w \int_c^d h(y) L(y) dy$$

where $h(y)$ is the depth of the fluid at y and $L(y)$ is the horizontal length of the region at y.

We will work a couple of problems in class.